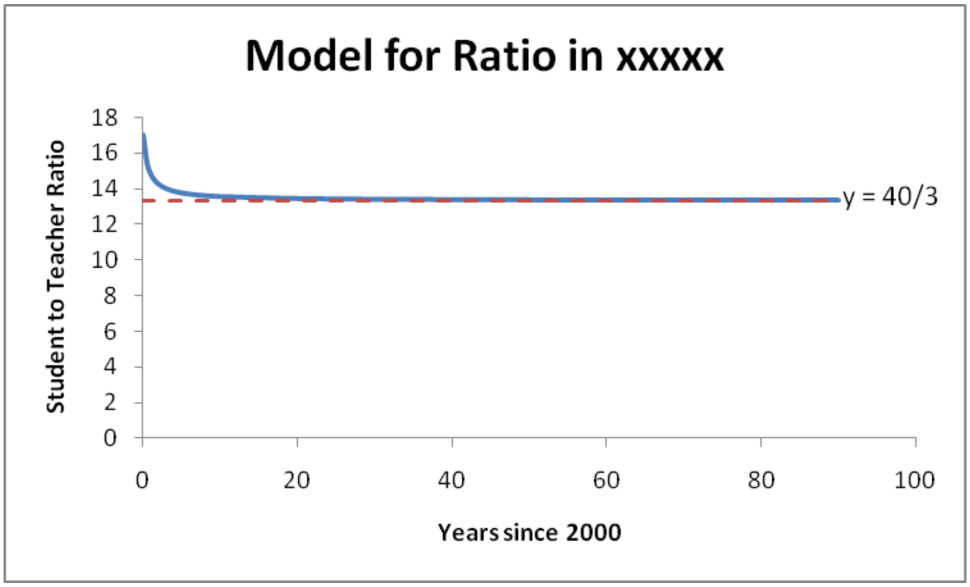
The goal of this technology assignment is to graph the function you found in Technology Assignment: Rational Model to model the student to teacher ratio for your state. You will produce a graph similar to the one below. Your graph should have the name of the state you have been assigned in place of xxxxx.



Your graph should have the following characteristics:

1. A graph of the model from Technology Assignment: Rational Model (you may modify this function from the earlier assignment if necessary).
2. A horizontal asymptote that shows the student to teacher ratio leveling off at a sensible level.
3. The horizontal asymptote should be labeled with its equation.
4. Each axis should be labeled.
5. The graph should have an appropriate title that includes your state.

Once you have created the graph in Excel, copy and paste the graph into a Word document. Include your name and the date in the document.

Follow the instructions in this handout to produce a graph like the one above for the state you were assigned in the project. We'll start by discussing limits and horizontal asymptotes. Then we'll look at how to create the graph in Excel. You'll turn in a Word document with the graph pasted into it.

Limits and Horizontal Asymptotes

Examining the graph above, you can see that the graph levels off at a vertical value of $y = \frac{40}{3}$. The farther you move along the graph to the right, the closer the y-values on the graph get to $\frac{40}{3}$. If R(t) represents the student to teacher ratio t years after 2000, we can use limits to express this relationship as

$$\lim_{t\to\infty} R(t) = \frac{40}{3}$$

This means that as t gets larger and larger, the ratio R(t) gets closer and closer to $\frac{40}{3}$. We can get a sense for this by examining a table in which t values get larger. Note that each row gives a larger value of t and the corresponding ratio. The ratios appear to be dropping and getting closer to a value of approximately 13.3333 which is the decimal equivalent of $\frac{40}{3}$ to four decimal places. We can estimate the value of the limit from the table, but to get an exact value (and the location of the horizontal asymptote) we need to evaluate this limit exactly.

t	R(t)
10	13.5625
100	13.35762
1000	13.33578
10000	13.33358
100000	13.33336
1000000	13.33334

Let's assume the graph above is the graph of a rational function

$$R(t) = \frac{20t+17}{1.5t+1}$$

and evaluate

$$\lim_{t\to\infty} \frac{20t+17}{1.5t+1}$$

We'll need the following rule to do limits at infinity:

For any positive real number,

$$\lim_{t\to\infty} \frac{1}{t^n} = 0$$

This rule states that as t becomes larger and larger without bound, the function $\frac{1}{t^n}$ approaches 0. This makes sense since for positive values of n, the denominator t^n grows larger so the fraction $\frac{1}{t^n}$ gets closer and closer to 0. This rule is also true as t decreases without bound. In other words,

$$\lim_{t\to-\infty} \frac{1}{t^n} = 0.$$

Example – Evaluate the limit $\lim_{t\to\infty} \frac{20t+17}{1.5t+1}$.

To evaluate a limit at infinity for a rational function, we must divide the numerator and denominator by the largest power on a variable that appears in the numerator. In this case, the highest power that appears in the denominator $1.5t+1$ is 1. This means we must divide each term in the numerator and denominator by t^1:

$$\lim_{t\to\infty}\frac{20t+17}{1.5t+1}=\lim_{t\to\infty}\frac{\frac{20t}{t}+\frac{17}{t}}{\frac{1.5t}{t}+\frac{1}{t}}$$

By dividing the top and the bottom by the same expression, we are making no change to the rational function. Simplifying the expression leads to

$$\begin{aligned}\lim_{t\to\infty}\frac{\frac{20t}{t}+\frac{17}{t}}{\frac{1.5t}{t}+\frac{1}{t}}&=\lim_{t\to\infty}\frac{20+\frac{17}{t}}{1.5+\frac{1}{t}}\\&=\frac{\lim\limits_{t\to\infty}20+17\lim\limits_{t\to\infty}\frac{1}{t}}{\lim\limits_{t\to\infty}1.5+\lim\limits_{t\to\infty}\frac{1}{t}} &&\text{Utilize rules for limits}\\&=\frac{20+0}{1.5+0} &&\lim_{t\to\infty}\frac{1}{t}=0\\&=\frac{40}{3}\end{aligned}$$

In the example above, the degree of the numerator and denominator are the same. The examples below illustrates what happens when the degree in the numerator or denominator are not the same.

Example – Evaluate the limit $\lim\limits_{x\to\infty}\frac{2x-3}{x^2+1}$.

To evaluate this limit, notice that the highest bower that appears on a variable in the denominator is 2. This means that we'll divide each term in the numerator and denominator by x^2:

$$\begin{aligned}\lim_{x\to\infty}\frac{2x-3}{x^2+1}&=\lim_{x\to\infty}\frac{\frac{2x}{x^2}-\frac{3}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}}\\&=\lim_{x\to\infty}\frac{\frac{2}{x}-\frac{3}{x^2}}{1+\frac{1}{x^2}}\end{aligned}$$

Using the rules for limits, we can break this limit into smaller problems,

$$\lim_{x\to\infty}\frac{\frac{2}{x}-\frac{3}{x^2}}{1+\frac{1}{x^2}}=\frac{2\lim\limits_{x\to\infty}\frac{1}{x}-3\lim\limits_{x\to\infty}\frac{1}{x^2}}{\lim\limits_{x\to\infty}1+\lim\limits_{x\to\infty}\frac{1}{x^2}}$$
$$=\frac{0-0}{1+0}$$
$$=0$$

$$\lim_{x\to\infty}\frac{1}{x^n}=0 \text{ for } n=1,2$$

In general, when the degree in a rational function is higher in the denominator, the limit will be zero since the bottom grows much faster than the top.

Example – Evaluate the limit $\lim\limits_{x\to\infty}\frac{x^3-x^2}{x+2}$.

In this limit, the degree on the numerator is higher than the degree in the denominator. As in the previous two examples, we'll divide each term in the rational function by the variable raised to the highest power in the denominator, x:

$$\lim_{x\to\infty}\frac{x^3-x^2}{x+2}=\lim_{x\to\infty}\frac{\frac{x^3}{x}-\frac{x^2}{x}}{\frac{x}{x}+\frac{2}{x}}$$
$$=\lim_{x\to\infty}\frac{x^2-x}{1+\frac{2}{x}}$$
$$=\frac{\lim\limits_{x\to\infty}x^2-\lim\limits_{x\to\infty}x}{\lim\limits_{x\to\infty}1+\lim\limits_{x\to\infty}\frac{2}{x}}$$
$$=\frac{\lim\limits_{x\to\infty}x^2-\lim\limits_{x\to\infty}x}{1+0}$$

$$\lim_{x\to\infty}\frac{1}{x}=0$$

The denominator approaches a value of 1 as x gets very large. However, the numerator will get huge due to the presence of the term $\lim\limits_{x\to\infty}x^2$. Because of this term, the fraction will grow without bound. The limit does not exist and we write

$$\lim_{x\to\infty}\frac{x^3-x^2}{x+2}=\infty$$

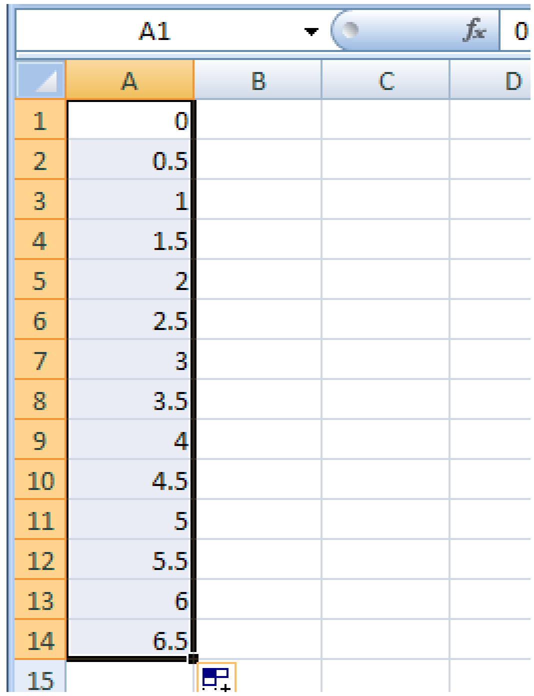
to symbolize this behavior. If the fraction had decreased without bound, we would have used -∞ to indicate the behavior.

To make the graph shown earlier in this handout, we'll need to create a table in several parts. Once the table is created, we'll graph that table. For this demonstration, I'll work with the rational function

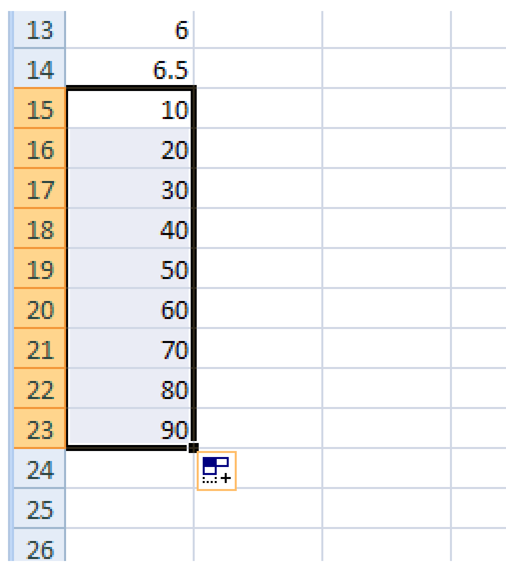
$$R(t) = \frac{20t + 17}{1.5t + 1}$$

You should use the rational function you created for the student to teacher ratio in your state.

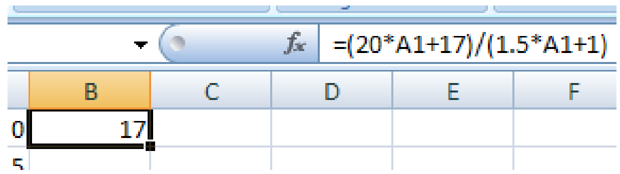
1. Open Excel.
2. In the first column, start a table at 0 in increments of 0.5. Fill the column to a value of about 6 or 7. These are the initial t-values that will help us to capture the sharp curve in the left part of the graph.



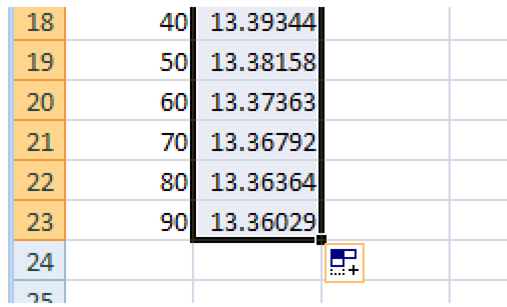
3. Since our graph extends to 90, we don't want to make our table all the way to 90 in increments of 0.5. In the next cell in column A (in this case A15), place a 10.
4. In the cell below, place a 20.
5. Select these two cells and fill more of the column so that you get numbers from 10 to 90 in increments of 10. Now column A has t-values from 0 to 6.5 in increments of 0.5 and then t-values from 10 to 90 in increments of 10.



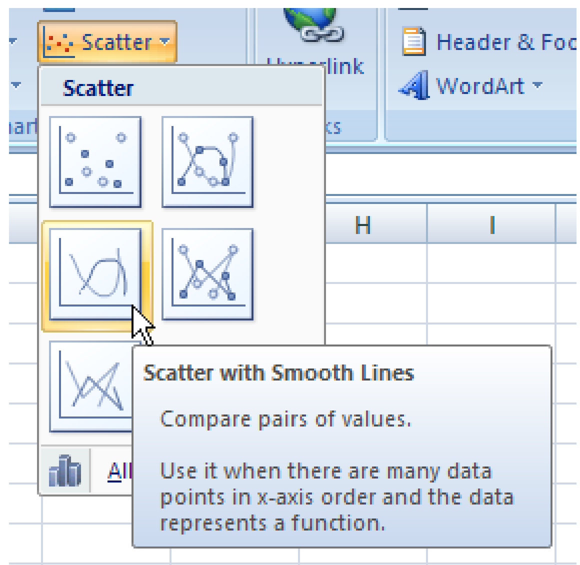
6. In cell B1, enter the formula for the function for your state's student to teacher ratio.
7. Press Enter on your keyboard to calculate the value of the function.



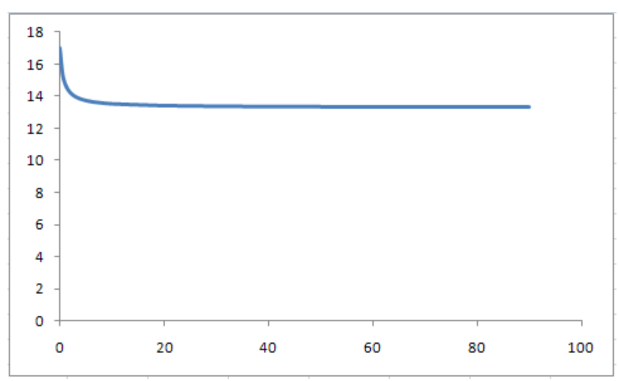
8. Select cell B1 and fill the rest of the column with y-values corresponding to each t-value in column A.



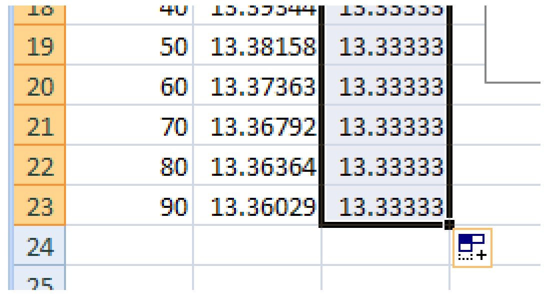
9. These two columns are the t- and y- values for the graph. Select these two columns.
10. From the Insert tab, select Scatter and then Scatter with Smooth Lines. This will place a graph of the values in the worksheet.



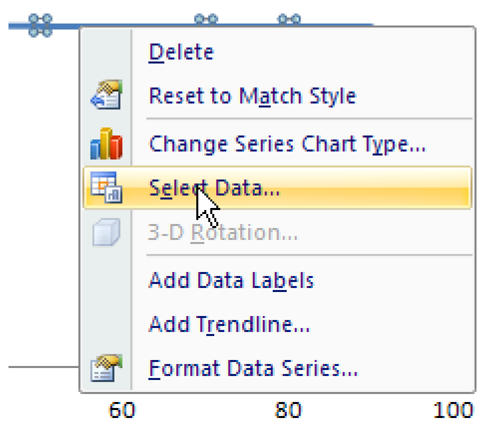
11. Click on the legend "Series 1" to select it and delete it.
12. Click on one of the gridlines to select it and delete it.



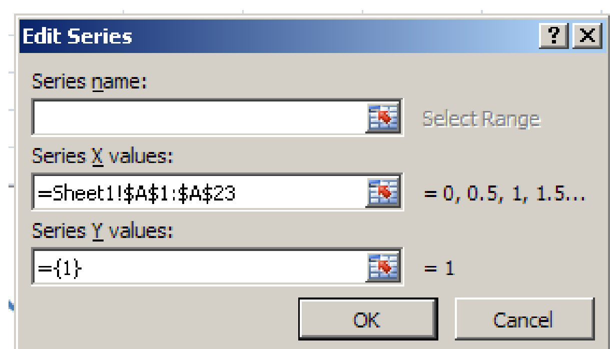
13. As hoped, the function graphed here levels off at a horizontal asymptote. Let's place the horizontal asymptote on the graph. Earlier I found that the horizontal asymptote is at $y = \frac{40}{3}$. To graph this line, we need to create a column for it in the table. In cell C1, type = 40/3.
14. Fill column C with this value by selecting C1 and stretching the fill handle.



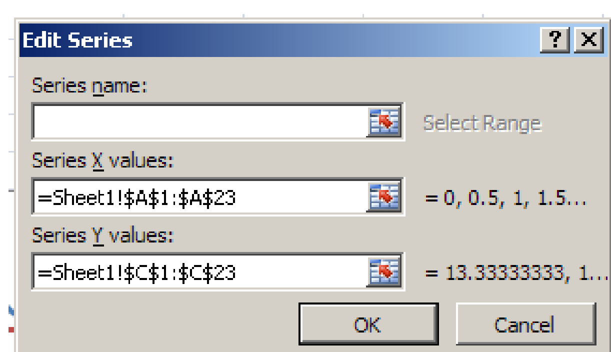
15. Click on the curve in the graph to select it.
16. Right mouse click on the graph and choose Select Data…
17. In the Select Data Source box that appears, choose Add.

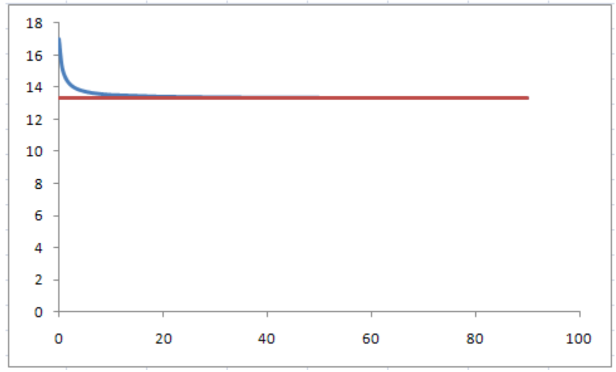


18. In the Edit Series box, click in space below the Series X values.
19. Select cell A1 and drag your mouse to further select all of the t-values in column A.



20. In the area labeled Series Y values, delete any text that appears.
21. While still in that area, use your mouse to select the y-values for the horizontal asymptote. What you see pictured to the right means that you'll graph values from cell A1 through A23 horizontally and values from C1 through C23 vertically.



<table>
<tr>
<td>22. The horizontal asymptote will be added to your graph. Notice how the asymptote covers up much of the function's graph. In our next steps, we'll modify the graph of the asymptote to be a thinner dashed line.</td>
<td>
</td>
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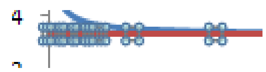
<td>23. Carefully click on the asymptote to select it. You'll see the points become highlighted to indicate which graph you are on. You may need to hunt around a bit to select the asymptote and not the function.</td>
<td>
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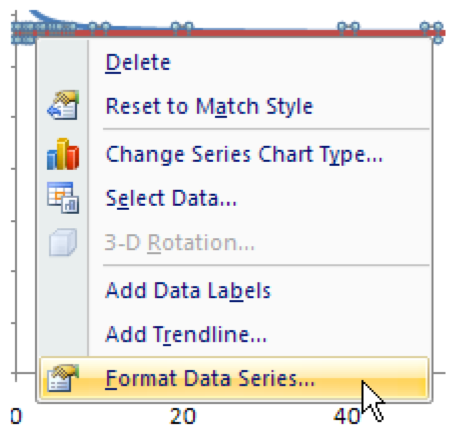
<td>24. Right mouse click on the graph.
25. Select Format Data Series…</td>
<td>
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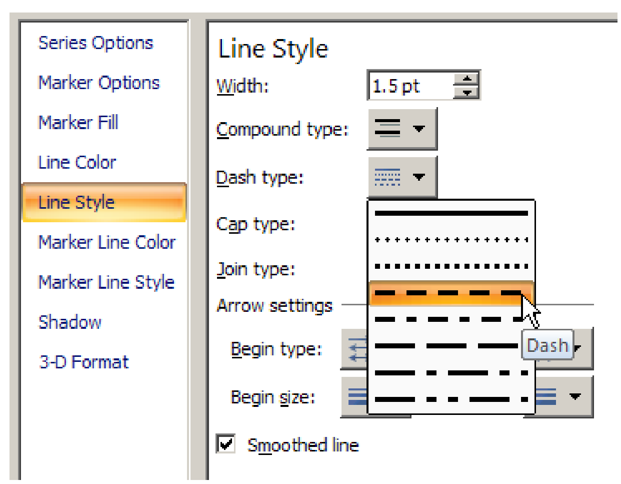
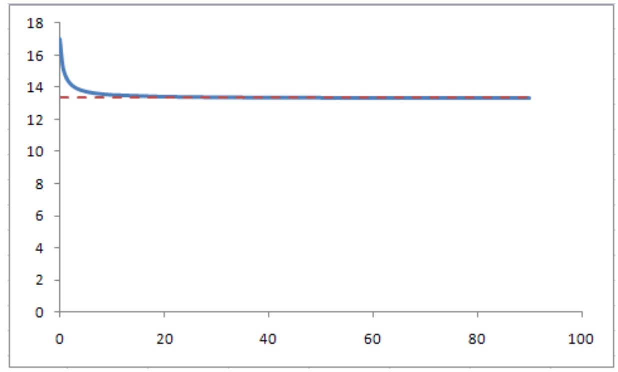
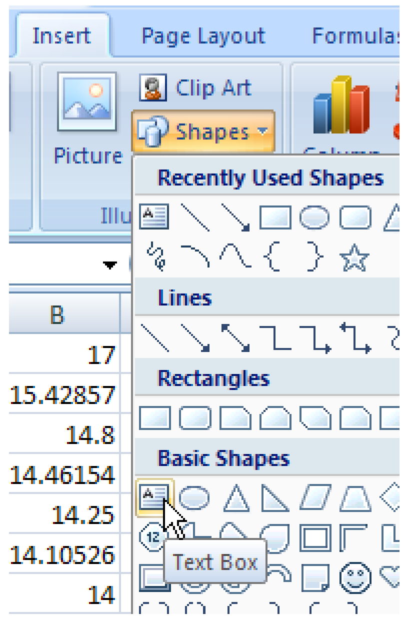
<td>26. The Format Data Series box allow you to change the characteristics of the selected curve. Choose Line Style along the left side of this box.
27. Change the Width of the curve to 1.5 pt.
28. Change the Dash type to Dash.</td>
<td>
</td>
</tr>
</table>

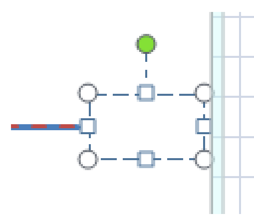
29. Now both graphs are much easier to distinguish.



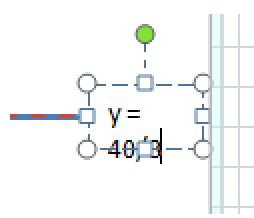
30. In this series of steps, we'll add a label y = 40/3 to the graph. Click on the edge of the graph to select it.
31. From the Insert tab, select Shapes.
32. Select Text Box.



33. Left mouse click and drag your mouse to create a text box near the right end of the graph.



34. Type the equation of the asymptote in the box. If the equation won't fit on a single line, you'll need to stretch the size of the box using the square handles along the border of the text box.



35. Finally, label each of the axes like you did in Tech Assignment 2. You may also choose to change the window a bit.
36. Copy and paste the graph of the model in your state in a Word document. In this document, make sure you include your name, class and the date. Make sure you remember to save this document as well as the Excel file you have just created.